ELECTROMAGNETIC WAVES

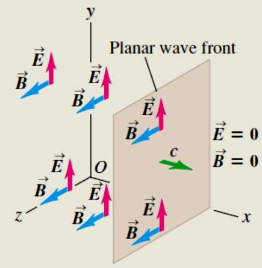
Some theory recal:

Maxwell's equations and electromagnetic waves: Maxwell's equations predict the existence of electromagnetic waves that propagate in vacuum at the speed of light c. The electromagnetic spectrum covers frequencies from at least 1 to 10^{24} Hz and a correspondingly broad range of wavelengths. Visible light, with wavelengths from 380 to 750 nm, is only a very small part of this spectrum. In a plane wave, $\vec{E}$ and $\vec{B}$ are uniform over any plane perpendicular to the propagation direction. Faraday's law and Ampere's law both give relationships between the magnitudes of $\vec{E}$ and $\vec{B}$; requiring both of these relationships to be satisfied gives an expression for c in terms of ϵ_0 and μ_0. Electromagnetic waves are transverse; the $\vec{E}$ and $\vec{B}$ fields are perpendicular to the direction of propagation and to each other. The direction of propagation is the direction of $\vec{E} \times \vec{B}$.

$$E = cB \quad (32.4)$$

$$B = \epsilon_0 \mu_0 c E \quad (32.8)$$

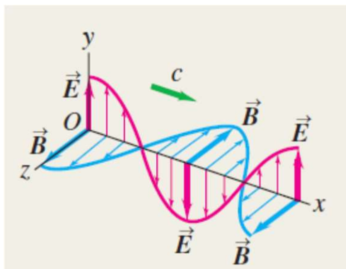
$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \quad (32.9)$$



Sinusoidal electromagnetic waves:

$$\vec{E}(x, t) = \hat{\jmath} E_{max} \cos(kx - \omega t)$$
$$\vec{B}(x, t) = \hat{k} B_{max} \cos(kx - \omega t)$$

$$E_{max} = c B_{max}$$



Electromagnetic waves in matter: When an electromagnetic wave travels through a dielectric, the wave speed is less than the speed of light in vacuum c.

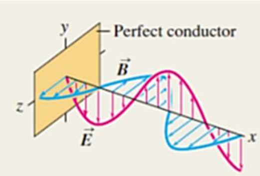
$$v = \frac{1}{\sqrt{\epsilon\mu}} = \frac{1}{\sqrt{KK_m}} \frac{1}{\sqrt{\epsilon_0\mu_0}} = \frac{c}{\sqrt{KK_m}}$$

Energy and momentum in electromagnetic waves: The energy flow rate (power per unit area) in an electromagnetic wave in vacuum is given by the Poynting vector $\vec{S}$. The magnitude of the time-averaged value of the Poynting vector is called the intensity I of the wave.

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

$$I = S_{av} = \frac{E_{max} B_{max}}{2\mu_0} = \frac{{E_{max}}^2}{2\mu_0 c} = \frac{1}{2}\sqrt{\frac{\epsilon_0}{\mu_0}}\,{E_{max}}^2 = \frac{1}{2}\epsilon_0 c\,{E_{max}}^2$$

Standing electromagnetic waves: If a perfect reflecting surface is placed at $x = 0$, the incident and reflected waves form a standing wave. Nodal planes for $\vec{E}$ occur at $kx = 0, \pi, 2\pi, \ldots$, and nodal planes for $\vec{B}$ at $kx = \pi/2, 3\pi/2, 5\pi/2, \ldots$. At each point, the sinusoidal variations of $\vec{E}$ and $\vec{B}$ with time are 90° out of phase.



ELECTROMAGNETIC WAVES

1/ Give several examples of electromagnetic waves that are encountered in everyday life. How are they all alike? How do they differ?

R:
Examples: *Radio waves, Visible light, UV light, X-Rays, Gamma-rays, Microwaves*
Common features: *they all propagate with the speed of light in vacuum c=3·10^{8}m/s, theu do not need a medium to propagate.*
Differences: *the frequency f and the wavelength are different (λ=c/f)*

2/ Sometimes neon signs located near a powerful radio station are seen to glow faintly at night, even though they are not turned on. What is happening?

R: *The electromagnetic waves emitted by the radio station are composed by time varying electric fields **E**(t) and magnetic field **B**(t). This field can ionize the neon gas in the tubes and then the ions undergo oscillations following the orientation of the electric field E(t). From the Maxwell equations, any charge in accelerated motion emits electromagnetic waves. This will lead to a continuous spectrum of the faintly emitted light. On the other hand, accelerated ions by the same electric field component of the electromagnetic wave may collide one to each-other and get to excited energetical state (see later the Bohr model of the atom). Then, when they will fall down to the fundamental state, electromagnetic waves (light) will be emitted. This will lead to a discrete component of the fainted emitted light.*

3/ Most automobiles have vertical antennas for receiving radio broadcasts. Explain what this tells you about the direction of polarization of in the radio waves used in broadcasting.

R: *The electrons from the metal composing the antenna wire should vertically oscillate in the vertical antenna due to the electromagnetic field of the electromagnetic wave. It means that the electric field has a vertical orientation. This means that the polarization of the electromagnetic waves (radio waves) is vertical.*

4/ A plane sinusoidal electromagnetic wave in air has a wavelength of 3.84 cm and an E-field amplitude of 1.35 V/m. (a) What is the frequency? (b) What is the B-field amplitude? (c) What is the intensity? (d) What average force does this radiation exert on a totally absorbing surface with area 0.240 m^2 perpendicular to the direction of propagation?

IDENTIFY: $c = f\lambda$. $E_{\max} = cB_{\max}$. $I = \frac{1}{2}\epsilon_0 c E_{\max}^2$. For a totally absorbing surface the radiation pressure is $\frac{I}{c}$.

SET UP: The wave speed in air is $c = 3.00\times10^8$ m/s.

EXECUTE: **(a)** $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8 \text{ m/s}}{3.84\times10^{-2} \text{ m}} = 7.81\times10^9 \text{ Hz}$

(b) $B_{\max} = \dfrac{E_{\max}}{c} = \dfrac{1.35 \text{ V/m}}{3.00\times10^8 \text{ m/s}} = 4.50\times10^{-9} \text{ T}$

(c) $I = \frac{1}{2}\epsilon_0 c E_{\max}^2 = \frac{1}{2}(8.854\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8 \text{ m/s})(1.35 \text{ V/m})^2 = 2.42\times10^{-3} \text{ W/m}^2$

(d) $F = (\text{pressure})A = \dfrac{IA}{c} = \dfrac{(2.42\times10^{-3} \text{ W/m}^2)(0.240 \text{ m}^2)}{3.00\times10^8 \text{ m/s}} = 1.94\times10^{-12} \text{ N}$

EVALUATE: The intensity depends only on the amplitudes of the electric and magnetic fields and is independent of the wavelength of the light.

5/ A small helium–neon laser emits red visible light with a power of 4.60 mW in a beam that has a diameter of 2.50 mm. (a) What are the amplitudes of the electric and magnetic fields of the light? (b) What are the average energy densities associated with the electric field and with the magnetic field? (c) What is the total energy contained in a 1.00-m length of the beam?

(a)

EXECUTE: The intensity is power per unit area: $I = \dfrac{P}{A} = \dfrac{4.60\times10^{-3}\ \text{W}}{\pi(1.25\times10^{-3}\ \text{m})^2} = 937\ \text{W/m}^2$.

$I = \dfrac{E_{\max}^2}{2\mu_0 c}$, so $E_{\max} = \sqrt{2\mu_0 cI}$. $E_{\max} = \sqrt{2(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(2.998\times10^8\ \text{m/s})(937\ \text{W/m}^2)} = 840\ \text{V/m}$.

$B_{\max} = \dfrac{E_{\max}}{c} = \dfrac{840\ \text{V/m}}{2.998\times10^8\ \text{m/s}} = 2.80\times10^{-6}\ \text{T}$.

EVALUATE: The magnetic field amplitude is quite small compared to laboratory fields.

(b)

EXECUTE: The energy density in the electric field is $u_E = \frac{1}{2}\epsilon_0 E^2$. $E = E_{\max}\cos(kx-\omega t)$ and the averag

value of $\cos^2(kx-\omega t)$ is $\frac{1}{2}$. The average energy density in the electric field then is

$u_{E,\text{av}} = \frac{1}{4}\epsilon_0 E_{\max}^2 = \frac{1}{4}(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(840\ \text{V/m})^2 = 1.56\times10^{-6}\ \text{J/m}^3$. The energy density in the magnetic field is $u_B = \dfrac{B^2}{2\mu_0}$. The average value is $u_{B,\text{av}} = \dfrac{B_{\max}^2}{4\mu_0} = \dfrac{(2.80\times10^{-6}\ \text{T})^2}{4(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})} = 1.56\times10^{-6}\ \text{J/m}^3$.

(c) IDENTIFY and SET UP: The total energy in this length of beam is the average energy density ($u_{\text{av}} = u_{E,\text{av}} + u_{B,\text{av}} = 3.12\times10^{-6}\ \text{J/m}^3$) times the volume of this part of the beam.

EXECUTE: $U = u_{\text{av}}LA = (3.12\times10^{-6}\ \text{J/m}^3)(1.00\ \text{m})\pi(1.25\times10^{-3}\ \text{m})^2 = 1.53\times10^{-11}\ \text{J}$.

EVALUATE: This quantity can also be calculated as the power output times the time it takes the light to travel $L = 1.00$ m: $U = P\left(\dfrac{L}{c}\right) = (4.60\times10^{-3}\ \text{W})\left(\dfrac{1.00\ \text{m}}{2.998\times10^8\ \text{m/s}}\right) = 1.53\times10^{-11}\ \text{J}$, which checks.

6/ The electric-field component of a sinusoidal electromagnetic wave traveling through a plastic cylinder is given by the equation:

$$E=(5.35\,V/m)\cos[(1.39.10^{7}\,rad/m)x-(3.02.10^{15}\,rad/s)t].$$

(a) Find the frequency, wavelength, and speed of this wave in the plastic. (b) What is the index of refraction of the plastic? (c) Assuming that the amplitude of the electric field does not change, write a comparable equation for the electric field if the light is traveling in air instead of in plastic.

IDENTIFY: We know the electric field in the plastic.

SET UP: The general wave function for the electric field is $E = E_{\max}\cos(kx-\omega t)$. $f = \dfrac{\omega}{2\pi}$, $\lambda = \dfrac{2\pi}{k}$, $v = f\lambda$ and $v = \dfrac{c}{n}$.

EXECUTE: **(a)** By comparing the equation for E to the general form, we have $\omega = 3.02\times10^{15}$ rad/s and $k = 1.39\times10^7$ rad/m. $f = \dfrac{\omega}{2\pi} = 4.81\times10^{14}$ Hz. $\lambda = \dfrac{2\pi}{k} = 4.52\times10^{-7}\ \text{m} = 452\ \text{nm}$.

$v = f\lambda = 2.17\times10^8\ \text{m/s}$.

(b) $n = \frac{c}{v} = \frac{3.00 \times 10^8 \text{ m/s}}{2.17 \times 10^8 \text{ m/s}} = 1.38.$

(c) In air, $\omega = 3.02 \times 10^{15}$ rad/s, the same as in the plastic. $\lambda_0 = \lambda n = (4.52 \times 10^{-7} \text{ m})(1.38) = 6.24 \times 10^{-7}$ m, so $k = \frac{2\pi}{\lambda} = 1.01 \times 10^7$ rad/m. The equation for E in air is

$$E = (535 \text{ V/m}) \cos\left[(1.01 \times 10^7 \text{ rad/m})x - (3.02 \times 10^{15} \text{ rad/s})t\right].$$

7/ A circular loop of wire has radius 7.50 cm. A sinusoidal electromagnetic plane wave traveling in air passes through the loop, with the direction of the magnetic field of the wave perpendicular to the plane of the loop. The intensity of the wave at the location of the loop is 0.0195W/m^2, and the wavelength of the wave is 6.90 m. What is the maximum emf induced in the loop?

IDENTIFY: The changing magnetic field of the electromagnetic wave produces a changing flux through the wire loop, which induces an emf in the loop.

SET UP: $\Phi_B = B\pi r^2 = \pi r^2 B_{\max} \cos(kx - \omega t)$, taking x for the direction of propagation of the wave. Faraday's law says $|\mathcal{E}| = \left|\frac{d\Phi_B}{dt}\right|$. The intensity of the wave is $I = \frac{E_{\max} B_{\max}}{2\mu_0} = \frac{c}{2\mu_0} B_{\max}^2$, and $f = \frac{c}{\lambda}$.

EXECUTE: $|\mathcal{E}| = \left|\frac{d\Phi_B}{dt}\right| = \omega B_{\max} \sin(kx - \omega t)\pi r^2$. $|\mathcal{E}|_{\max} = 2\pi f B_{\max} \pi r^2$.

$f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{6.90 \text{ m}} = 4.348 \times 10^7$ Hz. Solving $I = \frac{E_{\max} B_{\max}}{2\mu_0} = \frac{c}{2\mu_0} B_{\max}^2$ for $B_{\max}$ gives

$$B_{\max} = \sqrt{\frac{2\mu_0 I}{c}} = \sqrt{\frac{2(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(0.0195 \text{ W/m}^2)}{3.00 \times 10^8 \text{ m/s}}} = 1.278 \times 10^{-8} \text{ T}.$$

$$|\mathcal{E}|_{\max} = 2\pi(4.348 \times 10^7 \text{ Hz})(1.278 \times 10^{-8} \text{ T})\pi(0.075 \text{ m})^2 = 6.17 \times 10^{-2} \text{ V} = 61.7 \text{ mV}.$$

8/ In a certain experiment, a radio transmitter emits sinusoidal electromagnetic waves of frequency 110.0 MHz in opposite directions inside a narrow cavity with reflectors at both ends, causing a standing-wave pattern to occur. **(a)** How far apart are the nodal planes of the magnetic field? **(b)** If the standing-wave pattern is determined to be in its eighth harmonic, how long is the cavity?

IDENTIFY: The nodal planes are one-half wavelength apart.

SET UP: The nodal planes of B are at $x = \lambda/4, 3\lambda/4, 5\lambda/4, \ldots,$ which are $\lambda/2$ apart.

EXECUTE: **(a)** The wavelength is $\lambda = c/f = (3.000 \times 10^8 \text{ m/s})/(110.0 \times 10^6 \text{ Hz}) = 2.727$ m. So the nodal planes are at $(2.727 \text{ m})/2 = 1.364$ m apart.

(b) For the nodal planes of E, we have $\lambda_n = 2L/n$, so $L = n\lambda/2 = (8)(2.727 \text{ m})/2 = 10.91$ m.

EVALUATE: Because radiowaves have long wavelengths, the distances involved are easily measurable using ordinary metersticks.